Energy Efficient Control of Robots with Variable Stiffness Actuators ⋆

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Abstract: Variable stiffness actuators realize a particular class of actuators characterized by the property that the apparent output stiffness can be changed independently of the output position. This is feasible due to the presence of internal springs and internal actuated degrees of freedom. In this work, we establish a port-based model of variable stiffness actuators and we derive an energy efficient control strategy. In particular, when the variable stiffness actuator acts on a mechanical system, the internal degrees of freedom are used to achieve the desired nominal behavior and the internal springs are used as a potential energy buffer. The release of energy from the springs, as well as the apparent output stiffness, are regulated by control of the internal degrees of freedom. Simulation results on a robotic joint illustrate the effectiveness of the control strategy during the tracking of a periodic motion in presence of disturbances.

Keywords: Variable stiffness actuators, Energy efficient control, Mathematical models, Nonlinear analysis, Robotics

1. INTRODUCTION

The research interests and efforts in variable stiffness actuators are increasing in recent years due to their broad range of possible applications. The main characteristic property of a variable stiffness actuator is that the output stiffness can be varied independently from the output position, thanks to the presence of internal actuated degrees of freedom and internal springs. This means that, if the joints of a robot are actuated by means of this class of actuators, it is possible for the robot to perform different tasks while appearing more or less compliant.

Recently, various designs of variable stiffness actuators have been introduced, for example AMASC (Hurst, 2004), VSA (Tonietti, 2005), VS-Joint (Wolf, 2008) and MACCEPA (Vanderborght, 2009). All these actuators use a number of internal springs with a fixed elastic constant, and the output stiffness is varied by changing the configuration of some internal degrees of freedom. In the context of safe interaction, the mechanical compliance is controlled only if an unexpected collision occurs and it is used to reduce the impact force (De Luca, 2009).

Besides using the internal springs of a variable stiffness actuator solely to introduce a mechanical compliance to the joints for safety reasons (Bicchi, 2004), they can also be exploited to achieve more energy efficient actuation by storing negative work (Stramigioli, 2008) or to change the natural frequencies of the system to match the periodicity of the motion (Uemura, 2009). In recent work, we presented a novel energy efficient variable stiffness actuator, characterized by the property that the output position and output stiffness are decoupled on a mechanical level (Visser, 2010a). This property allows the internal springs to be used as buffers to temporarily store potential energy. For example, when a disturbance occurs, the springs can store the disturbance energy, which then can be reused to bring the robot back to the desired trajectory. In particular, this approach can have big advantages in trajectory tracking (Li and Horowitz, 1999; Duindam, 2004). In such applications, the desired joint trajectories do not depend on time, and hence the potential energy stored in the springs can be used efficiently. This approach can be beneficial to walking robots, where energy efficiency is of paramount concern and trajectory tracking controllers can improve the robustness (Duindam and Stramigioli, 2009).

Building on our previous work, in this paper we establish a formal, port-based mathematical model for the analysis and the control of variable stiffness actuators. The port-based framework not only provides valuable insights on energy flows between the internal actuators, the springs and the robot, but it is also a solid foundation for research on innovative control methods. With the aim of achieving energy efficiency, we derive a control architecture that uses the potential energy stored in the springs to actuate the robot, instead of supplying this energy by controlling the internal degrees of freedom. We demonstrate the effectiveness of the proposed controller for a one degree of freedom system, under influence of a disturbance. In particular, if a disturbance occurs, the corresponding energy is stored in the internal springs and used for actuation. The insights gained will form a basis for future work, in which we aim to develop energy efficient coordinated control methods for a robot with multiple degrees of freedom.

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2. PORT-BASED MODELING OF VARIABLE STIFFNESS ACTUATORS

In this Section, we intend to briefly recall the port-based generalized model of variable stiffness actuators, introduced in our previous work (Visser, 2010a,b), and to provide a more solid mathematical foundation, which is the basis for both the present paper and future research. The port-based framework gives valuable insights in the power flows between the controller, the variable stiffness actuator and the actuated system. Therefore, it realizes an appropriate tool for the analysis, modeling and control of systems in which energy efficiency is the main concern.

Without loss of generality, we assume that a variable stiffness actuator has the following properties:

- $n \geq 2$ internal degrees of freedom, denoted by $q \in \mathcal{Q}$, can be actuated;
- $m \geq 1$ springs, either linear or nonlinear, are internally present;
- the apparent output stiffness K of the actuator depends on both the configuration of the internal degrees of freedom and of the internal springs.

Since the aim of the model is to analyze the functional principle of a variable stiffness actuator, rather than evaluating the mechanical design, internal friction and inertias are neglected. The generic model of a variable stiffness actuator is depicted in Figure 1, using a bond graph representation. Each bond represents a power flow, defined positive in the direction of the half arrow and characterized by two power conjugate variables, called efforts e and flows f. If $\mathcal{F}$ is the linear space of admissible flows, then the dual space $\mathcal{E} := \mathcal{F}^*$ is the linear space of admissible efforts. For $e \in \mathcal{E}$ and $f \in \mathcal{F}$, the dual product $\langle e|f\rangle$ yields power. The Dirac structure $\mathcal{D}$ defines the interconnection of the bonds and, thus, defines how power flows from one bond to the others. The Dirac structure is power continuous, as follows from its formal definition (van der Schaft, 2000):

$$\mathcal{D} = \{\mathcal{D} \subset \mathcal{E} \times \mathcal{F} \mid \langle e|f\rangle = 0 \quad \forall (e, f) \in \mathcal{D}\}$$

The multidimensional $\mathbb{C}$-type *storage element* represents the springs in the device. Springs are characterized by the state $s \in \mathcal{S}$, i.e. their elongation or compression, and by the energy they store, described by the function $H : \mathcal{S} \to \mathbb{R}$. The effort e_s and flow f_s are, respectively, the forces generated by the springs and the rate of change of the states. The multidimensional *control port* is characterized by power conjugate variables τ and $\dot{q}$, i.e. the generalized forces that actuate the internal degrees of freedom and the generalized rate of change of the configuration variables. The *output port* is characterized by power conjugate variables F and $\dot{x}$, i.e. the generalized output force and the generalized output velocity, and, since it is assumed that the variable stiffness actuator is connected to a joint with one degree of freedom, it is one-dimensional. Depending on the type of actuator, the output force can be either a linear force or a torque and, correspondingly, the output position $x \in \mathcal{X}$ can either be a linear displacement or an angle. Note that the efforts τ and F do not depend on the flows $\dot{q}$ and $\dot{x}$ since a *power continuous transformation* between forces and velocities, called gyration, does not regularly exist in the mechanical domain.

The stiffness K at the output is given by

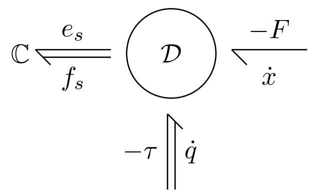

Fig. 1. Generalized representation of a variable stiffness actuator. The Dirac structure defines the interconnection between the different bonds and, therefore, how power is distributed among the ports. The multi-bonds allow any number of springs, i.e. the $\mathbb{C}$-element, and any number of external inputs $(\tau, \dot{q})$. The output port $(F, \dot{x})$ is one-dimensional and thus a single-bond.

$$K = \frac{\delta F}{\delta x}$$

where δF and δx denote infinitesimal changes.

Before entering into the details of the design of the energy based control law, we further analyze this model and derive the formal mathematical structure that gives more insight into the Dirac structure and its properties.

Following the arguments above, there exists a map

$$\Gamma_{(q,x)} : \mathcal{Q} \times \mathcal{X} \to \mathcal{S}$$

that defines how the configuration of the internal degrees of freedom $q \in \mathcal{Q}$ and the output position $x \in \mathcal{X}$ determine the states $s \in \mathcal{S}$ of the springs. The tangent map $\Gamma_{(q,x)*} : T\mathcal{Q} \times T\mathcal{X} \to T\mathcal{S}$ and cotangent map $\Gamma^*_{(q,x)} : T^*\mathcal{S} \to T^*\mathcal{Q} \times T^*\mathcal{X}$ are naturally defined (Nijmeijer and van der Schaft, 1990). The fiber bundle $\pi_u : \mathcal{U}_q \times \mathcal{U}_x \to \mathcal{Q} \times \mathcal{X}$ has fibers $\pi_u^{-1}(q, x)$ that denote the input spaces $U_q \times U_x$. Given an input $(u_q, u_x) \in U_q \times U_x$, the internal configuration q and the output position x are subject to the dynamics $G : \mathcal{U}_q \times \mathcal{U}_x \to T\mathcal{Q} \times T\mathcal{X}$ given by

$$\dot{q} = u_q, \quad \dot{x} = u_x \tag{1}$$

Equation (1) can be written as

$$\dot{q} = \sum_{i=1}^{n} v_{q,i} u_{q,i}, \quad \dot{x} = v_x u_x \tag{2}$$

where

$$\begin{aligned} v_{q,i} &= (\ \underbrace{0 \ \cdots \ 0}_{i-1 \text{ elements}} \ \ 1 \ \ \underbrace{0 \ \cdots \ 0}_{n-1 \text{ elements}}\ )^T \\ v_x &= 1 \end{aligned} \tag{3}$$

define constant input vector fields on $T\mathcal{Q} \times T\mathcal{X}$, i.e. the canonical basis for the tangent space. Since these dynamics are trivial, we allow abuse of notation and consider $\dot{q}$ and $\dot{x}$ as inputs to the system. These relations are summarized in the commutative diagram in Figure 2.

The dynamics of the variable stiffness actuator are

$$\begin{aligned} \dot{s} &= \Gamma_{(q,x)*}\ (\dot{q}, \dot{x}) \\ (\tau, F) &= \Gamma^*_{(q,x)}\ (\mathrm{d}H) \end{aligned} \tag{4}$$

where $\mathrm{d}H$ denotes the differential of the energy function H. We can see that flows are elements of the tangent spaces and efforts elements of the cotangent spaces. The Dirac structure is in the tangent maps $\Gamma_{q*} : T\mathcal{Q} \to T\mathcal{S}$, $\Gamma_{x*} : T\mathcal{X} \to T\mathcal{S}$ and the corresponding cotangent maps Γ^*_q, Γ^*_x. Via these maps, the velocities on $T_s\mathcal{S}$ at $s \in \mathcal{S}$ are

$$\begin{tikzcd}
T\mathcal{S} \arrow[d] & T\mathcal{Q}\times T\mathcal{X} \arrow[l, "\Gamma_{(q,x)*}"'] \arrow[d] & \mathcal{U}_q\times\mathcal{U}_x \arrow[l, "G"'] \arrow[dl, "\pi_u"] \\
\mathcal{S} & \mathcal{Q}\times\mathcal{X} \arrow[l, "\Gamma_{(q,x)}"'] & \\
T^*\mathcal{S} \arrow[u] \arrow[r, "\Gamma^*_{(q,x)}"] & T^*\mathcal{Q}\times T^*\mathcal{X} \arrow[u] &
\end{tikzcd}$$

Fig. 2. Commutative diagram for a variable stiffness actuator. $\mathcal{Q}$ is the configuration manifold of the internal degrees of freedom, $\mathcal{S}$ of the state of the internal springs, and $\mathcal{X}$ of the output position.

defined by the velocities on $T_q\mathcal{Q}\times T_x\mathcal{X}$ at $(q,x)\in\mathcal{Q}\times\mathcal{X}$, and correspondingly the forces on $T_q^*\mathcal{Q}\times T_x^*\mathcal{X}$ by the forces on $T_s^*\mathcal{S}$, such that power continuity is preserved.

The rate of change of the energy stored in the springs is given by (Bullo and Lewis, 2004):

$$\dot{H} = \mathrm{d}H\ \dot{s} = \mathrm{d}H\ \Gamma_{(q,x)*}(\dot{q},\dot{x}) \tag{5}$$

From (4) and (5), it can be immediately observed how the output force and the energy stored in the mechanism are related to each other, and how the rate of change of the stored energy depends on the output ports. In particular, from (5) it follows that the energy stored in the springs does not change due to the control input if

$$\dot{q} \in \ker\Gamma_{q*} \tag{6}$$

where $\ker\Gamma_{q*}$ denotes the kernel of the map Γ_{q*}. Note that the existence of this kernel depends on the mechanical design of the variable stiffness actuator (Visser, 2010a).

3. ENERGY EFFICIENT CONTROL

In this Section, an energy efficient control law for variable stiffness actuators is derived. This means that, in order to achieve the energy efficiency of the controller, we intend to use the energy stored in the internal springs, if present, for the actuation of the robotic joint. First, we formulate the problem statements and, then, derive a solution that accomplishes the energy efficiency requirements. Moreover, since the fundamental demand of variable stiffness actuators is the capability of changing the output stiffness, we extend the control law with stiffness regulation.

3.1 Problem Statement

As depicted in Figure 1, the variable stiffness actuator is connected to a robotic joint with one degree of freedom. This connection is explicitly shown in Figure 3 by a 1-junction representing the power continuous connection

$$\dot{x} = y, \quad u = -F$$

where (u,y) is the *interaction port* of the joint. In particular, u denotes the input force (or torque) applied to the joint, and y its velocity. We assume that full state measurement is available, i.e. for both joint and variable stiffness actuator all positions and velocities can be measured.

We now design an energetically efficient control architecture for the variable stiffness actuator, by using both the internal springs and internal actuated degrees of freedom, so that the actuator is able to apply a desired force (or

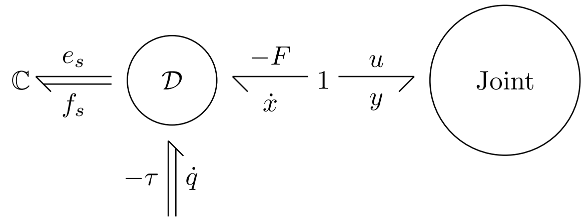

Fig. 3. Generalized representation of a variable stiffness actuator interconnected with a robotic joint.

torque) u_d to the joint and so that the output stiffness is regulated at a desired value. These goals are formally contained in the following problem statements.

Problem 3.1. *Consider a variable stiffness actuator, described by an energy function H and the dynamics (4), connected to a robotic joint by a power continuous connection. Assume that full state measurement is available. How can the control inputs $\dot{q}$ to the variable stiffness actuator be designed such that the desired joint input u_d is achieved in an energy efficient way?*

Problem 3.2. *How can the control inputs $\dot{q}$ to the variable stiffness actuator be designed such that a desired joint input u_d is achieved in an energy efficient way, while at the same time the output stiffness is controlled?*

In the solutions of these two problems, we aim to exploit the internal springs of the variable stiffness actuator as buffers for storing potential energy and to use this energy to actuate the joint. In the remainder of the Section, we first derive a control law that solves Problem 3.1 in a nominal case, i.e., without considering energy efficiency. Then, we focus on refining the control law by efficiently using any potential energy stored in the springs to reduce the energy supply via the control port. Finally, we derive the stiffness control law for Problem 3.2.

3.2 Output Force Control

Before proceeding with the formulation of the control law, it is necessary to highlight that, if a desired input force (torque) u_d is required at the robotic joint, also the desired output force F_d of the actuator is known due to the power continuous connection. Moreover, from (4), the output force F, generated by the variable stiffness actuator, can be determined once the current values for q and x are given. This means that it is possible to find a control law $g_F(\cdot)$ such that the evolution in time of the force F is given by

$$\dot{F} = g_F(F_d, F)$$

as a function of the actual and the desired output force. We can then formulate a nominal control law as follows.

Lemma 3.3. (Nominal control). Consider a variable stiffness actuator described by an energy function H and the dynamics (4), connected to the joint of a robot through a power continuous connection. Let the vector valued function $V(q,x)$ be:

$$V(q,x) := (L_{v_{q,1}}F, \ldots, L_{v_{q,n}}F) \tag{7}$$

where $L_{v_{q,i}}F$ denotes the Lie-derivative of F along the vector field $v_{q,i}$. Define a subset $\mathcal{M}$ of $\mathcal{Q}\times\mathcal{X}$ as:

$$\mathcal{M} = \{(q,x)\in\mathcal{Q}\times\mathcal{X} \mid V(q,x)\neq 0\}$$

By using the control law $\dot{F} = g_F(F_d, F)$, the nominal control input

$$\dot{q}_n = V^+ \left(\dot{F} - (L_{v_x} F)\, \dot{x} \right) \tag{8}$$

where $^+$ denotes the Moore-Penrose pseudo inverse, solves Problem 3.1 for (q, x) in $\mathcal{M}$. $\diamond$

Proof: Since we defined the exact one-form $\mathrm{d}H$ on $T_s^*\mathcal{S}$, we have that (τ, F) is an exact one-form on $T_q^*\mathcal{Q} \times T_x^*\mathcal{X}$, defined by the map $\Gamma^*_{(q,x)}$ (Nijmeijer and van der Schaft, 1990). Moreover, since the input vector fields, as defined in (2) and (3) are constant, the rate of change of the one-form (τ, F) is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}(\tau, F) = (\dot{q}_1 \ \cdots \ \dot{q}_n \ \dot{x}) \begin{pmatrix} L_{v_{q,1}}(\tau, F) \\ \vdots \\ L_{v_{q,n}}(\tau, F) \\ L_{v_x}(\tau, F) \end{pmatrix} \tag{9}$$

where we allowed some abuse of notation, as in (1). Since $\dim\,\mathcal{X} = 1$, from (9) it follows that:

$$\dot{F} = V\dot{q} + (L_{v_x} F)\, \dot{x} \tag{10}$$

with V given in (7). Finally, from (10), the nominal controller $\dot{q}_n$ in (8) follows.

Note that the restriction of the solution to $\mathcal{M}$ follows from the Moore-Penrose pseudo inverse for the full row-rank matrix V (Ben-Israel and Greville, 2003):

$$V^+ = V^T \left(V V^T \right)^{-1}$$

i.e., the pseudo inverse is only defined for (q, x) in $\mathcal{M}$. $\square$

3.3 Energy Efficient Control

The solution derived in Lemma 3.3 is not energy efficient, since the energy stored in the springs is not taken into account. If a disturbance is present on the output port, we intend to direct the corresponding energy to the springs so to use them as energy buffer. Since our goal is to obtain solutions that efficiently use energy stored in the springs, if there is energy stored in the springs, there is, in general, no need to supply more energy via the control port.

For a particular class of variable stiffness actuators, the solution (8) can be modified such that any potential energy stored in the springs is efficiently used to actuate the robot. This class of actuators has the property that output position and output stiffness are decoupled on a mechanical level and, therefore, the kernel in (6) exists. This means that the map Γ_{q*} is surjective onto $T_s\mathcal{S}$:

$$\text{rank}\ \Gamma_{q*} < n, \quad \forall (q, x) \in \mathcal{Q} \times \mathcal{X}$$

This property allows for a partitioning of the tangent space $T_q\mathcal{Q}$, i.e. the tangent space to $\mathcal{Q}$ at q, as:

$$T_q\mathcal{Q} = \ker \Gamma_{q*} \oplus D$$

where $\oplus$ is the direct sum and D is such that the tangent space $T_q\mathcal{Q}$ is complete. In particular, since $T_q\mathcal{Q}$ is a $\mathbb{R}$-vector space (Bullo and Lewis, 2004), we require D to be orthogonal to $\ker \Gamma_{q*}$ in the Euclidean sense. If we denote by k the dimension of $\ker \Gamma_{q*}$, i.e. $\dim\ \ker T_q\mathcal{Q} = k$, $1 \le k < n$, and since $\dim\ T_q\mathcal{Q} = n$, it follows that $\dim\ D = n - k$.

From (6) it was observed that, if the control input $\dot{q} \in \ker \Gamma_{q*}$, then the energy stored in the springs does not change due to the control input. Hence, the solutions to Problem 3.1 given by (8) should be in $\ker \Gamma_{q*}$ or close to it when energy is stored in the springs. This can be achieved by defining on $T_q\mathcal{Q}$ an appropriate metric that weights the solutions given by the Moore-Penrose pseudo inverse (Ben-Israel and Greville, 2003). This argument is formalized in the following Lemma.

Lemma 3.4. (Energy efficient control). Consider a variable stiffness actuator described by an energy function H and the dynamics (4) and connected to the joint of a robot by a power continuous connection. Assume that the variable stiffness actuator satisfies the property

$$\dim\ \ker \Gamma_{q*} = k, \quad 1 \le k < n, \quad \forall (q, x) \in \mathcal{Q} \times \mathcal{X}$$

Take on $T_q\mathcal{Q}$ two sets of local coordinates, orthogonal in the Euclidean sense, denoted by $a^1 = (a^1_1, \ldots, a^1_k)$ and $a^2 = (a^2_1, \ldots, a^2_{n-k})$, satisfying

$$\begin{aligned} \ker \Gamma_{q*} &= \text{span}\left\{ a^1 \right\} \\ D &= \text{span}\left\{ a^2 \right\} \end{aligned} \tag{11}$$

On $T_q\mathcal{Q}$, define a metric $\mathfrak{g}$, such that in the local coordinates (a^1, a^2) its components $[\mathfrak{g}]$ are given by

$$[\mathfrak{g}] = \begin{bmatrix} I_k & 0 \\ 0 & \alpha\, I_{n-k} \end{bmatrix} \tag{12}$$

with I_k and I_{n-k} the identity matrix of dimension k and $n-k$ respectively, and $\alpha : \mathcal{S} \to \mathbb{R}^+$ a positive definite function realizing a measure for the amount of energy stored in the springs.

Then, for (q, x) in the subset $\mathcal{M}$, the control input

$$\dot{q}_e = V^\sharp \left(\dot{F} - (L_{v_x} F)\, \dot{x} \right) \tag{13}$$

where $^\sharp$ denotes the Moore-Penrose pseudo inverse with respect to the metric $\mathfrak{g}$ defined in (12), solves Problem 3.1 in an energy efficient way by exploiting the energy stored in the springs. $\diamond$

Proof: Using the partitioning (11), the solution (8) can be expressed into components that are either in $\ker \Gamma_{q*}$ or outside, i.e. in D. Figure 4 depicts a two dimensional example, in which $\left(\frac{\partial}{\partial q_1}, \frac{\partial}{\partial q_2} \right)$ denotes the canonical basis for $T_q\mathcal{Q}$, the one-dimensional spaces $\ker \Gamma_{q*}$ and D are spanned by the vectors a^1 and a^2, respectively. By choosing the metric $\mathfrak{g}$ as proposed in (12), the components of the solution $\dot{q}_e$ in D are weighted by α. In particular, since α is chosen such that it is proportional to the energy stored in the springs, the component in D is smaller when there is more energy available in the springs. Hence, from (5) it follows that, when there is more energy available in the springs, less energy is supplied via the control port. $\square$

3.4 Stiffness Control

The control law derived in Lemma 3.4 achieves control of the joint position, but does not control the apparent stiffness of the joint. However, in many applications it is desired that the apparent joint stiffness attains some specific value. Therefore, we extend the control law with stiffness control. In particular, because of the redundancy in the internal degrees of freedom, we can define a control input, to add to any solution of Problem 3.1, so that the stiffness is controlled independently from the generated

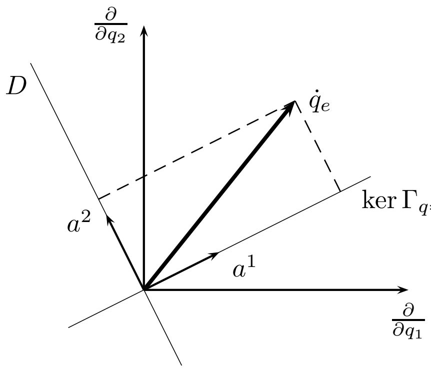

Fig. 4. Decomposition of the energy efficient control law $\dot{q}_e$ in (13) into components in $\ker \Gamma_{q*}$ and in D.

output force. This requires that the additional stiffness control $\dot{q}_{kV} \in \ker V$, so to not affect the control $\dot{q}_e$ in (13).

Under the assumption of full state measurement, the apparent output stiffness K of the actuator may be estimated. Given a desired output stiffness K_d, we can design a control law $g_K(\cdot)$ such that the desired rate of change of the stiffness is given by

$$\dot{K} = g_K(K_d, K)$$

If we model the stiffness K as a function on the configuration manifold, i.e. $K : \mathcal{Q} \times \mathcal{X} \to \mathbb{R}$, we can formulate a control law for the stiffness as follows.

Lemma 3.5. (Stiffness control). Define on $T_q\mathcal{Q}$ two sets of coordinates, denoted by b^1 and b^2, satisfying:

$$\ker V = \operatorname{span}\left\{b^1\right\}$$
$$T_q\mathcal{Q} = \operatorname{span}\left\{b^1, b^2\right\}$$

Determine a solution $\dot{q}_k$ that achieves the desired rate of change of the stiffness, i.e. a solution satisfying

$$\left(L_{v_{q,1}}K \;\cdots\; L_{v_{q,n}}K\right)\; \dot{q}_k = g_K(K_d, K)$$

where $L_{v_{q,i}}K$ denotes the Lie-derivative of K along $v_{q,i}$. Denote by $\dot{q}_{kV}$ the projection of $\dot{q}_k$ onto b^1. Then, the solution to Problem 3.2 is given by the control input

$$\dot{q}_k = \dot{q}_e + \dot{q}_{kV} \tag{14}$$

$\diamond$

Proof: The control input $\dot{q}_k$ is chosen such that the stiffness changes as desired, and by taking the projection onto $\ker V$, the stiffness is changed while Problem 3.1 is still solved. $\square$

Remark 3.6. By taking the projection onto $\ker V$, it is ensured that at all times Problem 3.1 is solved. However, it follows that, in general, $\dot{q}_{kV} \neq \dot{q}_k$, and thus that the stiffness does not change exactly as desired, but as close to desired as possible. $\triangleleft$

Remark 3.7. Since $\dot{q}_{kV}$ was not obtained with respect to the metric $\mathfrak{g}$ as defined in Lemma 3.4, the solution (14) is not necessarily energy efficient. In particular, the choice of $g_K(\cdot)$ determines the component outside $\ker \Gamma_{q*}$. $\triangleleft$

4. SIMULATION RESULTS

In this Section, we show the effectiveness of the control law derived in Section 3. This will be done by using a linear variable stiffness actuator design, presented in earlier work (Visser, 2010a), which satisfies the condition stated in Lemma 3.4, to actuate a joint. The experiment is as follows. The actuator moves a linear joint on a periodic motion following a sinusoidal trajectory with an amplitude of 10 cm at a frequency of 0.1 Hz. The force u_d needed to make the joint follow the trajectory is calculated using a PD-control law using the current and desired position and velocity. In the time interval $5 \leq t \leq 6$ s, the joint is subjected to a 2 N constant disturbance force. The same experiment is performed with each of the three presented control laws (8), (13), (14), and their performance is compared. In particular, it is investigated how each of the controllers handles the disturbance energy. The results are presented in Figures 5-7.

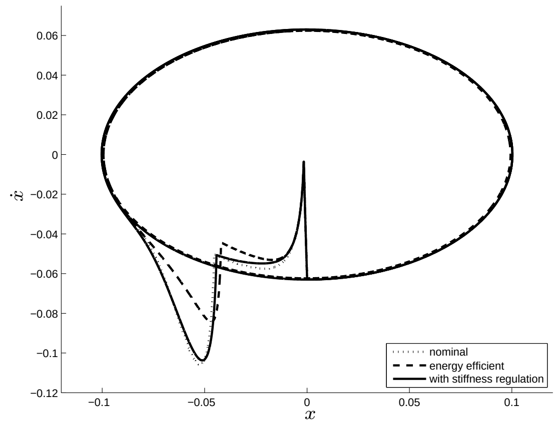

Fig. 5. Joint phase diagram - In a periodic motion, the disturbance is similarly corrected by each control law.

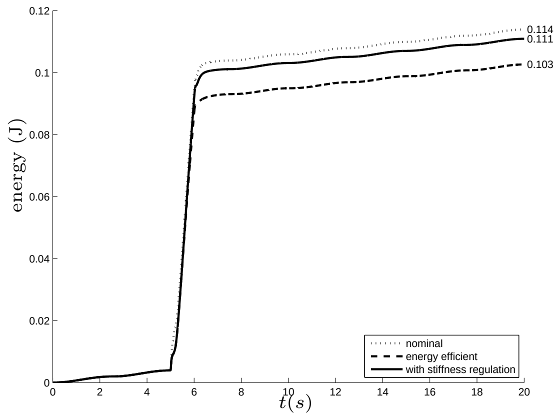

Fig. 6. Energy supplied via the control port - The energy efficient control law clearly achieves a reduction in supplied energy. However, when stiffness regulation is added, some of the gain in efficiency is lost.

Figure 5 shows the phase space trajectory of the joint. It can be seen that the response to the disturbance is similar for each of the controllers. Note that each controller required approximately the same amount of time to return to the desired trajectory. In Figure 6, the energy supplied via the control port of the variable stiffness actuator is plotted. In particular, the absolute power flow through the control port is integrated, to make explicit that negative work is lost. From the numerical values, it can be seen that the energy efficient control law indeed achieves a sig-

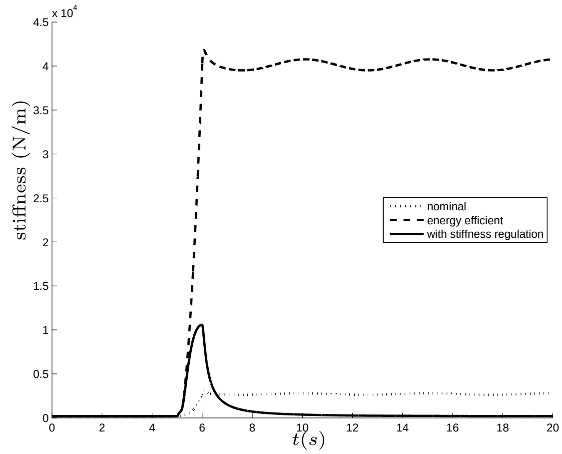

Fig. 7. Output stiffness - Adding stiffness regulation to the control law ensures that the stiffness of the joint is kept close to a desired value of 200 N/m.

nificant reduction in energy consumption (approximately 9.6% with respect to the nominal control law). When stiffness regulation is added, the reduction in energy consumption is less (approximately 2.6%), as was expected. From Figure 7, the added benefit of stiffness regulation can be seen. Initially, the output stiffness is kept to a desired value of 200 N/m. When the disturbance occurs, the output stiffness increases, because the internal springs store the disturbance energy. The increase in stiffness might be an undesiderable side effect in some applications, e.g. in human-robot interaction, and thus the potential benefit of adding stiffness regulation is illustrated. However, at the same time, it is illustrated that energy efficient control and regulating stiffness are contradicting goals.

5. CONCLUSIONS AND FUTURE WORK

In this work, we presented an energy efficient control method for variable stiffness actuators. In addition, a stiffness regulation control was implemented, with the aim of maintaining a desired output stiffness. Simulation results illustrate the effectiveness of the proposed method. In particular, it was shown that the energy efficient control law indeed achieves a significant reduction in the energy supplied via the control port of the variable stiffness actuator by reusing energy stored in the springs. Adding stiffness regulation reduces the energy efficiency, but the controller still performs better than the nominal controller. It was found that the strict time dependency of the reference trajectory in the simulation can sometimes cause the controllers to perform poorly. This is due to the inherent oscillatory behaviour of the springs. We believe that, in limit cycle trajectory tracking applications, the advantages of the proposed control strategy will become more apparent. Since in such trajectory tracking application time is no longer restrictive, the controller may perform better by taking a state dependent response to the disturbance.

Future work will focus on controlling multiple degree of freedom systems under influence of significant disturbances, where the energy storing capabilities of the springs are more useful. The aim is to control the joints in a coordinated way, and come to an energy efficient disturbance correction. In particular, in the control of walking robots, the energy losses, associated with the impacts of the feet, can be reduced using our proposed control method.

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